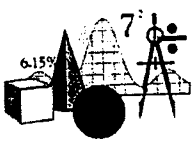
THE CATHOLIC UNIVERSITY OF AMERICA



First Annual CUA
Newton Mathematics Competition
2005

Isaac Newton (1642-1727)

Individual Competition

Problem 1. Find the product of the real roots of the equation

$$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}.$$

Problem 2. Compute

$$(\log_2 3)(\log_3 4)(\log_4 5)...(\log_{255} 256).$$

Problem 3. Show that

$$\arctan 1 + \arctan 2 + \arctan 3 = \pi.$$

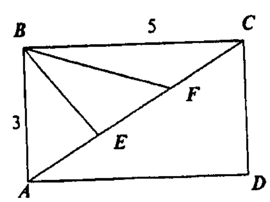
Problem 4. Simplify the sum

$$\frac{1}{1\times 2} + \frac{2}{1\times 2\times 3} + \frac{3}{1\times 2\times 3\times 4} + \cdots + \frac{n}{1\times 2\times 3\times \cdots \times n\times (n+1)}$$

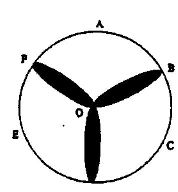
and show that it is < 1 for each positive integer n.

Problem 5. Find the area of a circle inscribed in a rhombus whose perimeter is 100 and whose longer diagonal has length 40.

Problem 6. The length of rectangle $ABCD$ is 5 inches and its width is 3 inches. Diagonal AC is divided into three equal segments by points E and F. Find the area of triangle BEF.



Problem 7. The circumference of a circle with radius a is divided into six equal arc lengths AB, AC etc. A set of 3 circular arcs of radius a are drawn, FOB, BOD and FOD, centered at A, C, and E respectively. Find the area of the shaded three-leaved rose.



Problem 8. The sides of the triangle $A'B'C'$ are longer than the corresponding sides of the triangle ABC ($A'B' > AB$, $A'C' > AC$, and $B'C' > BC$). Is it possible that the area of the triangle $A'B'C'$ is less than the area of the triangle ABC? Justify your answer.

Problem 9. Consider the collection of matches shown in the picture, see Figure 1 (each side of a small square is a match). Move exactly two matches in such a way that all matches in their new positions will form exactly 4 squares of the form (shown in the brackets below)

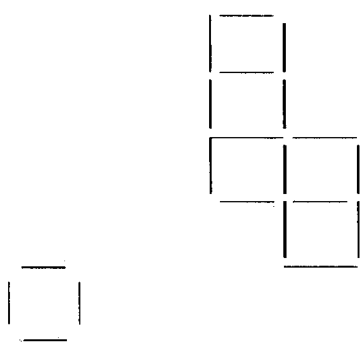


Figure 1:

and each match will be used in some square. Matches are not allowed to share the same position.

Problem 10. Prove that there is a natural number N which is divisible by 2007 and whose decimal representation ends with 2008.

Problem 11. Suppose that the admission office of a university has n applications. $n-1$ applicants can be business majors and only 1 applicant is permitted to be a math major. The admission office decided the following strategy: Applicants form a circle. A position is marked on this circle. In a clockwise direction the applicant standing next to this marked position, and those in every other spot after that leaves the circle immediately and is admitted to the business school. The process is continued until exactly one applicant is left, who becomes a math major. **Question.** Where should an applicant stand to achieve the goal of being a math major?